Math 221: Test 3 Review Sheet

This document tells you everything that you will need to know in order to do well on your first test. Each bullet is a requirement that you should ensure that you satisfy prior to taking the test. Not all bullets have problems following them, but these concepts are just as important.

- Know the definitions of all related terms to the various concepts covered in order to solve problems that use them. There will be no definitions question on this test.

5.1

- Represent addition/subtraction of integers using the chip/charged field and number line models.

Integer Addition and Subtraction: Represent the following problems using the chip/charged field model and the number line model.

1. $^-5 + 2 = {}^-3$
2. $^-2 + {}^-4 = {}^-6$
3. $3 + {}^-1 = 2$
4. $4 + {}^-4 = 0$
5. $2 - 6 = {}^-4$
6. $^-3 - {}^-5 = 2$
7. $^-2 - 2 = {}^-4$
8. $3 - {}^-1 = 4$
9. $0 - {}^-5 = 5$

5.2

- Represent multiplication of integers using the chip/charged field and number line models.

Integer Multiplication: Represent the following problems using the chip/charged field model and the number line model.

1. $3 \times 2 = 6$
2. $^-3 \times 2 = {}^-6$
3. $3 \times {}^-2 = {}^-6$
4. $^-3 \times {}^-2 = 6$
5. $0 \times {}^-1 = 0$
6. $4 \times 0 = 0$

6.1

- Build equivalent fractions using the Fundamental Law of Fractions.
- Write fractions in simplest form.

Building and Simplifying Fractions: Write the following fractions in simplest form, then build two more equivalent fractions from the simplified form.

1. $\frac{3}{12}$
2. $\frac{26}{52}$
3. $\frac{42}{60}$
4. $\frac{252}{78}$
5. $\frac{616}{96}$
6. $\frac{2662}{165}$

- Find whether two fractions are equivalent by using the methods simplification, finding a least common denominator, and cross multiplication.
- Find which of two fractions is larger by using the methods of finding a least common denominator and cross multiplication.

Comparing Fractions: Fill in the blank with <, >, or =. You need to be able to do so by simplification (for equality), least common denominator, and cross multiplication.

1. $\frac{1}{3}$ ___ $\frac{2}{5}$
2. $\frac{16}{20}$ ___ $\frac{8}{11}$
3. $\frac{4}{30}$ ___ $\frac{18}{135}$
4. $\frac{18}{35}$ ___ $\frac{1}{2}$
5. $\frac{7}{11}$ ___ $\frac{12}{23}$
6. $\frac{47}{111}$ ___ $\frac{9}{23}$

- Prove why cross multiplication works.

Proofs 1: Prove all of the following theorems.

1. Two fractions $\frac{a}{b}$ and $\frac{c}{d}$ are equivalent if and only if $ad = bc$.
2. If a, b, c, and d are integers with $b, d > 0$, then $\frac{a}{b} > \frac{c}{d}$ if and only if $ad > bc$.
3. If a, b, c, and d are integers with $b, d > 0$, then $\frac{a}{b} < \frac{c}{d}$ if and only if $ad < bc$.

<u>6.2</u>

- Draw a figure to represent addition/subtraction of fractions with the same denominators.

Fraction Models 1: Draw a diagram to solve the following addition/subtraction problems.

1. $\frac{1}{2} + \frac{1}{2}$
2. $\frac{2}{4} + \frac{1}{4}$
3. $\frac{3}{6} + \frac{1}{6}$
4. $\frac{2}{2} - \frac{1}{2}$
5. $\frac{2}{4} - \frac{1}{4}$
6. $\frac{3}{6} - \frac{1}{6}$

- Add or subtract fractions quickly using the "cross multiplication" theorems, then simplify.
- Add/subtract fractions with different denominators by using the least common denominator.

Adding/Subtracting Fractions: Solve the following addition/subtraction problems using the LCD and the cross multiplication theorem. Write your answers in simplest form.

1. $\frac{1}{4} + \frac{1}{6}$
2. $\frac{1}{3} + \frac{1}{6}$
3. $\frac{5}{18} + \frac{5}{27}$
4. $\frac{5}{72} + \frac{1}{30}$
5. $\frac{78}{104} + \frac{35}{91}$
6. $\frac{182}{588} + \frac{165}{693}$
7. $\frac{1}{4} - \frac{1}{6}$
8. $\frac{1}{3} - \frac{1}{6}$
9. $\frac{5}{18} - \frac{5}{27}$
10. $\frac{5}{72} - \frac{1}{30}$
11. $\frac{78}{104} - \frac{35}{91}$
12. $\frac{182}{588} - \frac{165}{693}$

- Convert between mixed numbers and improper fractions.
- Add/subtract mixed numbers without converting them to improper fractions.

Adding/Subtracting Mixed Numbers: Solve the following addition/subtraction problems without converting the mixed numbers to improper fractions. Write your answer as a correctly defined mixed number in simplest form.

1. $2\frac{1}{2} + 1\frac{3}{4}$
2. $3\frac{3}{8} + \frac{5}{8}$
3. $5\frac{7}{10} + 2\frac{8}{15}$
4. $2\frac{1}{2} - 1\frac{3}{4}$
5. $3\frac{3}{8} - \frac{5}{8}$
6. $5\frac{1}{10} - 2\frac{8}{15}$

- Prove the theorems referenced above.

Proofs 2: Prove all of the following theorems.

1. If $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers, then $\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$
2. If $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers, then $\frac{a}{b} - \frac{c}{d} = \frac{ad - bc}{bd}$

6.3

- Draw a figure to represent multiplication of fractions.

Fraction Models 2: Draw a diagram to solve the following multiplication problems.

1. $\frac{3}{4} \times \frac{1}{2}$
2. $\frac{2}{5} \times \frac{2}{3}$
3. $\frac{5}{6} \times \frac{4}{5}$

- Multiply fractions by simplifying as much as possible before performing the multiplication.

Multiplying Fractions: Solve the following multiplication problems by simplifying the problem as much as possible, then multiplying. Your final answer should be in simplest form without any further simplifications.

1. $\frac{3}{2} \times \frac{2}{15}$
2. $\frac{3}{14} \times \frac{70}{20}$
3. $\frac{21}{28} \times \frac{40}{66}$

- Multiply mixed numbers by either FOIL method (optional) or converting to improper fractions.

Multiplying Mixed Numbers: Solve the following multiplication problems by FOIL and by converting the mixed numbers to improper fractions. Write your final answer as a correctly defined mixed number in simplest form.

1. $1\frac{1}{2} \times 2\frac{1}{2}$
2. $2\frac{2}{3} \times 1\frac{1}{3}$
3. $6\frac{1}{4} \times 3\frac{3}{5}$

- Divide fractions by using Keep Change Flip with an explanation.

Dividing Fractions: Solve the following division problems by using Keep Change Flip with an explanation. Practice all 3 explanations we used in class if possible. Write your answers in simplest form.

1. $\frac{3}{8} \div \frac{8}{3}$
2. $\frac{5}{12} \div \frac{3}{4}$
3. $\frac{30}{35} \div \frac{99}{28}$

- Determine whether a given set has the closure, commutative, associative, identity, or inverse properties over multiplication.

  Properties of Multiplication: Determine whether the following sets have the closure, commutative, associative, identity, and/or inverse properties over multiplication. Explain each of your answers.

  1. $\{0, 1, 4, 9, 16, ...\}$
  2. $\left\{..., \frac{1}{4}, \frac{1}{3}, \frac{1}{2}, 2, 3, 4, ...\right\}$
  3. $\left\{\frac{a}{b} \middle| \; a, b \in \mathbb{N}\right\}$

- Give an example of the zero multiplication and distributive properties.

<u>7.1</u>

- Write a decimal in an expanded fraction form or as a whole number plus a single fraction over the largest power of 10 necessary.
- Write in words how to read a decimal.
- Identify the places held by a digit in a decimal.

  Decimal Properties: For each decimal, (a) write the fraction in expanded form and as a whole number plus a fraction over a single common denominator, (b) write in words how to read this decimal, (c) identify the digit in the indicated place values.

  1. 12.035
(units, hundredths)
  2. 0.1586
(tenths, ten-thousandths)
  3. 920.74
(hundreds, thousandths)

- Explain why zeros at the end of a decimal don't matter.
- Write a terminating decimal as a fraction.

  Decimals to Fractions: Convert the following decimals to simplified fractions.

  1. 0.625
  2. 0.5625
  3. 0.0175

- Determine if a fraction can be written as a terminating decimal, and if so, write it in that form without using division.

  Fractions to Decimals: Convert the following fractions to decimals. Check first if it is possible to write the fraction as a terminating decimal, and if so, do so by getting a power of 10 in the denominator, not long division.

  1. $\frac{6}{25}$
  2. $\frac{2}{9}$
  3. $\frac{13}{40}$
  4. $\frac{57}{75}$
  5. $\frac{21}{99}$
  6. $\frac{101}{144}$

- State the theorem regarding when a fraction can be written as a terminating decimal.

<u>7.2</u>

- Add, subtract, multiply, and divide decimals using standard algorithms.
- Explain why we choose our decimal place where we do in multiplication of decimals.
- Explain why we add a decimal point and a zero when we divide an integer by an integer and don't have a zero remainder.

- Explain why we add additional zeros when we are dividing a decimal by an integer and don't have a zero remainder.
- Explain why we can move the decimal place of the dividend and divisor an equal number of times in order to divide a decimal by a decimal.

Operations on Decimals: Solve the following problems using the standard algorithm. Leave your answers as a decimal, and rounded to the nearest hundredth if necessary.

1. $1.237 + 3.249$
2. $0.007 + 15.1937$
3. $5.214 - 0.763$
4. $3.34 - 1.3491$
5. 3.22×5.7
6. 40.001×8.91
7. $122.5 \div 16$
8. $659 \div 5.7$
9. $0.001 \div 8.91$
10. Using problems 5 and 6 as an example, explain why you chose to put your decimal place in that position and why that position works.
11. Using problem 8 as an example, explain why you add a ".0" to your number after you get a non-zero remainder and then add additional zeros when you still don't get a non-zero remainder.
12. Using problems 8 and 9 as an example, explain why you can move the decimal place over the same number of places in the dividend and the divisor to make an equivalent problem.

- Round decimals to a given place value.

Rounding Decimals: Round the following decimals to the indicated place value.

1. 1.8721 (tenths)
2. 0.8254 (hundredths)
3. 12.7996 (thousandths)

<u>7.3a</u>

- Convert fractions to repeating decimals using long division. (See Fractions to Decimals Above)
- Determine which of two decimals is larger, including repeating decimals.

Comparing Decimals: Fill in the blank with <, >, or =.

1. 1.142 ___ 1.14
2. $1.\overline{59}$ ___ $1.\overline{590}$
3. $0.\overline{63}$ ___ $0.\overline{637}$

Helpful Rules/Formulas:

1. (Additive Inverse Property) For any integer a, there is a value ^-a such that $^-a + a = 0$ and $a + {^-a} = 0$.
2. Addition with the chip/charged field model is done by drawing both numbers in and destroying paired positives and negatives.
3. Subtraction with the chip/charged field model is done by drawing the first number and removing the second number. We often have to add addition pairs of positives and negatives to do the calculations.
4. For addition and subtraction on the number line, we start by standing on the first number. We face right (positive direction) if we are adding and left (negative direction) if we are subtracting. We then move the number of steps given by the absolute value of the second number. If the second number is positive, we move forward; if it is negative, we move backwards.
5. Multiplication of integers $a \times b$ is defined to be what we have if we make $|a|$ groups containing b, then take the opposite if a is negative. If a is 0, the product is defined to be 0.
6. Multiplication with the chip/charged field model is done by drawing $|a|$ groups containing b, then changing the signs of all elements if a is negative. If a is 0, we would just draw a circle with no groups.
7. Multiplication with the number line model is done by first standing on 0. We then face left if a is negative and right if a is positive. Finally, we move $|a|$ steps of size $|b|$. If b is positive, the steps are forwards, and if b is negative, the steps are backwards.
8. Every integer is a rational number since it can be written as itself over 1.
9. Fundamental Law of Fractions: If $\frac{a}{b}$ is any fraction and n is a nonzero integer, then $\frac{a}{b} = \frac{an}{bn}$.
10. Two fractions $\frac{a}{b}$ and $\frac{c}{d}$ are equivalent if and only if $ad = bc$.
11. If a, b, c, d are integers with $b, d > 0$, then $\frac{a}{b} > \frac{c}{d}$ if and only if $ad > bc$.
12. If a, b, c, d are integers with $b, d > 0$, then $\frac{a}{b} < \frac{c}{d}$ if and only if $ad < bc$.
13. To add/subtract fractions with the same denominator, we add/subtract the numerators and keep the same denominator.
14. To add/subtract fractions with different denominators, we use the fundamental law of fractions to get a common denominator (LCD ideally), then add the new fractions.
15. Quick addition that usually requires extra simplification: $\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$
16. Quick subtraction that usually requires extra simplification: $\frac{a}{b} - \frac{c}{d} = \frac{ad - bc}{bd}$
17. To change a mixed number to an improper fraction, multiply the denominator times the whole number, then add the numerator. This value is the numerator of your new fraction, and we keep the same denominator.
18. To change an improper fraction, use long division until a remainder is reached. The whole number part is the quotient, and the fraction part is the remainder over the divisor.
19. To add mixed numbers, add the whole numbers, add the fractions, and rewrite if necessary.
20. To subtract mixed numbers, regroup from the first number to make an improper fraction if necessary. Then subtract the fractions and whole numbers.
21. We read decimals by stating the whole number, "and" for the decimal point, and then reading the decimal number as the fraction over a common denominator that it represents.
22. To write a fraction as a decimal, simply write what is being said when the decimal is read, then simplify the resulting fraction.
23. To convert a fraction to a terminating decimal (when possible), first simplify the fraction, then multiply its numerator and denominator by the appropriate power of 2 or 5 that will make the denominator a power of 10. Finally, write it as a decimal with the same number of decimals places as the exponent on the 10.
24. A rational number $\frac{a}{b}$ in simplest form can be written as a terminating decimal if and only if the prime

factorization of the denominator contains no primes other than 2 or 5.

25. To write a fraction as a repeating decimal, we perform the long division until we find the repeating pattern.
26. To compare two decimals, we use the following criteria: (1) if the whole number is bigger, that decimal number is bigger; (2) if the whole numbers are equal and the tenths digit is bigger (putting placeholder 0's if necessary), that decimal number is bigger; (3) continue this process until a location where the digits are different is found. If all the digits are the same, the decimal numbers are equal.
27. To compare repeating decimals, write enough iterations to make the comparison. If the repeating parts and the digits before it are the same, the repeating decimal numbers are equal.